The reasonable agreement between the values of  $\psi$  obtained from the condensation and suction experiments indicates that the simple division of the shear force into two components friction drag and momentum—is a valid approach.

## 5. CONCLUSIONS

The use of a Blasius-type expression for the shear stress at the interface of a turbulently flowing vapour and a laminar condensate film, applied in Nusselt's theory of laminar film condensation, cannot account for measured condensation rates with high vapour velocities.

There is considerable experimental evidence, both from condensation experiments and from experiments with flow in porous tubes, to suggest that condensation rates can be adequately predicted if the shear stress is taken to be equal to the product of the momentum loss coefficient  $\psi$ , the mean vapour velocity and the condensation rate per unit area. Although  $\psi$  varies with Reynolds number, condensation rate and x/D, a value of 0.75 is probably sufficiently accurate for most design calculations.

It has often been suggested that in counterflow, condensation rates would be lower than those occurring with zero vapour velocity because of the resulting thickening of the condensate film. This is not so, because the film becomes turbulent.

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# AMBIENT TEMPERATURE STRATIFICATION EFFECTS IN LAMINAR FREE CONVECTION

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(Received 6 February 1973)

#### NOMENCLATURE

$a_1, a_2, a_3,$	dimensionless constants;
g,	gravitational acceleration;
m, n,	dimensionless constants;
Τ.	temperature;
х,	distance along plate surface.

# Greek symbols

 $\beta$ , coefficient of thermal expansion :

v, kinematic viscosity.

Subscripts

- w, wall value;
- $\infty$ , ambient value.

# **INTRODUCTION**

IN A RECENT article Yang *et al.* [1] presented results of a similarity solution for laminar free convection from a nonisothermal vertical flat plate with a temperature stratified ambient environment. In addition to the Prandtl number, the similarity equations included two other parameters which are related to the wall and ambient temperature variations. The purpose of this note is to show the relation of these two parameters to dimensionless groups which are characteristic of free convection and thermal stratification. A simple equation is given that allows a quick calculation of the effect of a temperature stratified ambient environment on free convection boundary layer heat transfer.

## ANALYSIS

Starting with the usual free convection boundary-layer equations [1] and using the similarity technique of Yang [2], a solution is obtained if the wall and ambient temperature distributions are given by:

$$T_{w} - a_{3} = \frac{v^{2}}{g\beta} \left(\frac{a_{2}}{4n} + 1\right) a_{1} \left(\frac{4x}{a_{1}}\right)^{n}$$
(1)

$$T_{\infty} - a_3 = \frac{v^2}{g\beta} \left(\frac{a_2}{4n}\right) a_1 \left(\frac{4x}{a_1}\right)^n \tag{2}$$

where  $a_1$ ,  $a_2$ ,  $a_3$  and *n* are constants. Yang [1] has made the substitution  $a_2 = 4nm$  where *m* is just another constant. Subtracting equation (2) from (1) and combining the result

with the derivative of equation (2), yields:

$$\frac{a_2}{4} = nm = \frac{\frac{g\beta x^4}{v^2} \frac{dT_{\infty}}{dx}}{\frac{g\beta x^3}{v^2} (T_w - T_{\infty})}.$$
 (3)

The denominator of the right-hand side is recognized as the Grashof number and the numerator is a new dimensionless group that is a result of the ambient temperature stratification. While equations (1) and (2) determine the actual temperature distributions, equation (3) is useful in that it shows the relation of the parameters of the problem to dimensionless groups. To determine the effect of ambient temperature variation, one has only to fix the axial position and make a reasonable estimate of the wall-ambient temperature difference and the ambient temperature gradient. Equation (3) can then be used to determine the magnitude of  $a_2$ , or for the case of an isothermal plate (m = -1), the magnitude of n.

The particular case of free convection from an isothermal vertical plate in non-isothermal surroundings has been previously studied by Cheesewright [3]. From his results (for air), one may deduce that the heat transfer rate is a linear function of the parameter n for |n| < 0.4. A comparison of his numerical solution for n = -0.15 with a solution for constant ambient temperature [4] shows a 10 per cent increase in the heat transfer rate. In free convection experiments, the condition of constant ambient temperature (n = 0) is not always achieved [3], and it then becomes necessary to know the importance of the effect of the ambient temperature stratificaton.

Since *n* is not easily calculated from equation (2), equation (3) may be used to give an estimate of the magnitude of this parameter. As an illustrative example, consider a typical free convection experiment in air where there is a 20<sup>o</sup> temperature difference between the isothermal vertical surface and the ambient fluid. At an axial distance of 1.5 ft let the ambient temperature gradient be  $2^{\circ}F/ft$ , not an unreasonable estimate especially for experiments in closed cavities. Equation (3) then gives n = -0.15, and the heat transfer results can be expected to be in error by 10 per cent.

# CONCLUSIONS

A simple relation, equation (3), is given that allows the calculation of the effect of ambient temperature stratification in laminar free convection heat transfer. For a given ambient temperature variation, stratification effects may be reduced by making measurements at small axial positions, or by increasing the temperature difference between the vertical surface and the ambient fluid.

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